ARTICLE IN PRESS
JID: JARAP [m5G;January 20, 2018;8:48]
Annual Reviews in Control 000 (2018) 1–2
Contents lists available at ScienceDirect
Annual Reviews in Control
journal homepage: www.elsevier.com/locate/arcontrol

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Rudolf E. Kalman's quest for algebraic characterizations of positivity

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ARTICLE INFO

Article history:
Received 22 December 2017
Accepted 13 January 2018
Available online xxx

Keywords:
Rudolf E. Kalman

ABSTRACT

The present paper is a tribute to Professor Rudolf Emil Kalman, father of Mathematical System Theory and a towering figure in the field of Control and Dynamical Systems. Amongst his seminal contributions was a series of results and insights into the role of positivity in System Theory and in Control Engineering. The paper contains a collection of reminiscences by the author together with brief technical references that touch upon the unparalleled influence of Professor Kalman on this topic.

1. The early years

The miracle years for Rudolf Kalman began in 1957, one year before he submitted his doctoral dissertation on the subject of sampled data systems. A constant stream of original ideas began pouring out, ideas that were to transform the field of control, educate and inspire generations of theorists, and enable transformative new technologies. Chief among those was the Kalman filter, published in Kalman (1960), a most impactful contribution that is ubiquitous in today's technological advances, from guidance and navigation systems, cellphones, industrial controls to radar technology and vision systems. But of no lesser importance were his insights and parallel contributions to algebraic structure of dynamical systems, optimization, sampled data controls, and stability theory. The latter subject is intrinsically connected with positivity and passivity, concepts that were central to his thinking throughout his life.

His first paper on "Physical and mathematical mechanisms of instability in nonlinear automatic control," published in Kalman (1957), became very influential in the subsequent development of absolute stability theory (see Liberzon, 2006) and the topic never ceased to excite his interest and creativity. Shortly afterwards, in 1963, he published his work on Lyapunov functions for the Lur'e problem, (Kalman, 1963) inspired by Yakubovich (1962). This work brought to life the so-called Kalman–Yacoubovich–Popov (KYP) lemma that highlighted the centrality of passivity theory and was a harbinger of a great many developments that took place in the subsequent years until the present (Iwasaki & Hara, 2005; Rantzer, 1996). The KYP lemma proved the entry point for linear matrix inequalities and convex optimization in control theory (Willems, 1971) and of key importance in filtering, stochastic realizations, control design, circuit theory as well as in addressing Kalman's far reaching question on the inverse regulator problem (Kalman, 1964).

Moving forward, Kalman explored this circle of ideas in providing characterizations of stability with respect to more general algebraic domains (Kalman, 1965; Kalman, 1969) via conditions that capture positivity in a suitable sense. When reading these works, the converse question of characterizing domains via linear matrix inequalities is inescapable.

2. Post 1970

A duality between quadratic regulator theory and stochastic realization theory, expanded upon and summarized in P. Faurre's 1972 dissertation (cf. Faurre, Clerget, & Germain, 1978), played a key role in Kalman's thought during the following years. Pursuing leads in early work by K. Löwner, Kalman sought to obtain an algebraic characterization of minimal stochastic partial realizations (unpublished notes by Kalman). An alternative form of the same problem amounts to realizing a circuit with a minimal number of passive elements (more accurately, minimal McMillan degree) based on the value of its impedance at specified complex frequencies. Potential leads were sought in invariants of linear quadratic problems – a subject of conversation between the late E. Bruce Lee and Kalman during the "1975 Birch Island Conference on Systems Theory" in Bruce's cabin (Fig. 1), and a problem that was ultimately solved in 1981 by one of Kalman's students, Pramod Khargonekar, in his PhD thesis.

The topic of "positive linear systems" remained a focus in his research proposals, motivated by applications to circuits and mod-

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https://doi.org/10.1016/j.arcontrol.2018.01.001
1367-5788/

Please cite this article as: T.T. Georgiou, Rudolf E. Kalman's quest for algebraic characterizations of positivity, Annual Reviews in Control (2018), https://doi.org/10.1016/j.arcontrol.2018.01.001

Fig. 1. E. Bruce Lee and Rudy Kalman at Birch Island in 1975. Picture courtesy of E.B. Lee.

Fig. 2. R.E. Kalman with T.T. Georgiou at Kalman's home in Gainesville, Florida on June 12, 2016. Picture taken by Mrs. Dina Kalman.

eling of random signals. However, paraphrasing his words, progress was frustratingly slow. In a 1976 final progress report to AFOSR he highlighted the apparent difficulties in dealing with the "interaction between positive and algebraic." At that point, in the report, he declared that "no further work on this topic is planned in the near future." But those who knew Kalman, knew better, that he never gives up. Soon afterwards in "Realization of covariance sequences," (Kalman, 1982), he sought an answer in the form of algebraic inequalities in terms of Schur parameters.

His insights pointed to computational algebraic geometry which, however, didn't seem to have reached a satisfactory stage at the time and for the sought purpose. The same seemed true for classical decision theory, in the style of Tarski and Seidenberg. Under his direction, I began working on my dissertation on this topic in 1981. Early on my dear friend Pramod Khargonekar joined me on this, and together we worked through the classical literature on the moment problem (Georgiou & Khargonekar, 1982). Yet the "interaction between positive and algebraic" seemed once again a roadblock towards the type of invariants that Kalman sought. One more year went by before I saw how to characterize realizations of a fixed McMillan degree by topological methods (see Georgiou, 1983; Georgiou, 1987a; Georgiou, 1987b). I recall with a mix of nostalgia my last years in Gainesville and all the ups and downs during the final stretch. My interactions with Kalman were points of inspiration as well as contention, due to differing views on the nature of the problem. While the sequel of Kalman's program on partial realization of covariance sequences proved especially fruitful, with transformative developments in modeling of time series, entropic regularization, and high resolution spectral analysis (see Georgiou, 1987a; Byrnes, Georgiou, & Lindquist, 2001; Byrnes, Georgiou, Lindquist, & Megretski, 2006; Byrnes & Lindquist, 1997; Byrnes, Lindquist, Gusev, & Matveev, 1995; Georgiou, 1999), Kalman's vision of an *algebraic* characterization was never fully accomplished. It is the sincere hope of the author that the present paper serves as a motivation and a challenge to others to attempt the next steps. Kalman's dream and quest live on (Fig. 2).

References

Byrnes, C. I., Georgiou, T. T., & Lindquist, A. (2001). A generalized entropy criterion for Nevanlinna–Pick interpolation with degree constraint. *IEEE Transactions on Automatic Control, 46*(6), 822–839.

Byrnes, C. I., Georgiou, T. T., Lindquist, A., & Megretski, A. (2006). Generalized interpolation in H_∞ with a complexity constraint. *Transactions of the American Mathematical Society, 358*(3), 965–987.

Byrnes, C. I., & Lindquist, A. (1997). On duality between filtering and interpolation. In *Systems and control in the twenty-first century* (pp. 101–136). Springer.

Byrnes, C. I., Lindquist, A., Gusev, S. V., & Matveev, A. S. (1995). A complete parameterization of all positive rational extensions of a covariance sequence. *IEEE Transactions on Automatic Control, 40*(11), 1841–1857.

Faurre, P., Clerget, M., & Germain, F. (1978). *Operateurs rationnels positifs.*. Dunod, Elsevier Scientific Publishing Co..

Georgiou, T. T. (1983). *Partial realization of covariance sequences*. University of Florida Ph.D. thesis..

Georgiou, T. T. (1987a). Realization of power spectra from partial covariance sequences. *IEEE Transactions on Acoustics, Speech, and Signal Processing, 35*(4), 438–449.

Georgiou, T. T. (1987b). A topological approach to Nevanlinna-Pick interpolation. *SIAM Journal on Mathematical Analysis, 18*(5), 1248–1260.

Georgiou, T. T. (1999). The interpolation problem with a degree constraint. *IEEE Transactions on Automatic Control, 44*(3), 631–635.

Georgiou, T. T., & Khargonekar, P. P. (1982). On the partial realization problem for covariance sequences. In *Proc. Princeton conference on information systems and sciences* (p. 181).

Iwasaki, T., & Hara, S. (2005). Generalized KYP lemma: Unified frequency domain inequalities with design applications. *IEEE Transactions on Automatic Control, 50*(1), 41–59.

Kalman, R. E. (1957). Physical and mathematical mechanisms of instability in nonlinear automatic control systems. *Transactions of ASME, 79*(3), 553–566.

Kalman, R. E. (1960). A new approach to linear filtering and prediction problems. *Journal of Basic Engineering, 82*(1), 35–45.

Kalman, R. E. (1963). Lyapunov functions for the problem of Lur'e in automatic control. *Proceedings of the National Academy of Sciences, 49*(2), 201–205.

Kalman, R. E. (1964). When is a linear control system optimal? *Journal of Basic Engineering, 86*(1), 51–60.

Kalman, R. E. (1965). On the Hermite-Fujiwara theorem in stability theory. *Quarterly of Applied Mathematics, 23*(3), 279–282.

Kalman, R. E. (1969). Algebraic characterization of polynomials whose zeros lie in certain algebraic domains. *Proceedings of the National Academy of Sciences, 64*(3), 818–823.

Kalman, R. E. (1982). Realization of covariance sequences. In *Toeplitz Centennial* (pp. 331–342). Springer.

Liberzon, M. (2006). Essays on the absolute stability theory. *Automation and Remote Control, 67*(10), 1610–1644.

Rantzer, A. (1996). On the Kalman–Yakubovich–Popov lemma. *Systems & Control Letters, 28*(1), 7–10.

Willems, J. (1971). Least squares stationary optimal control and the algebraic Riccati equation. *IEEE Transactions on Automatic Control, 16*(6), 621–634.

Yakubovich, V. A. (1962). The solution of certain matrix inequalities in automatic control theory. In *Soviet Math. dokl: 3* (pp. 620–623).

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