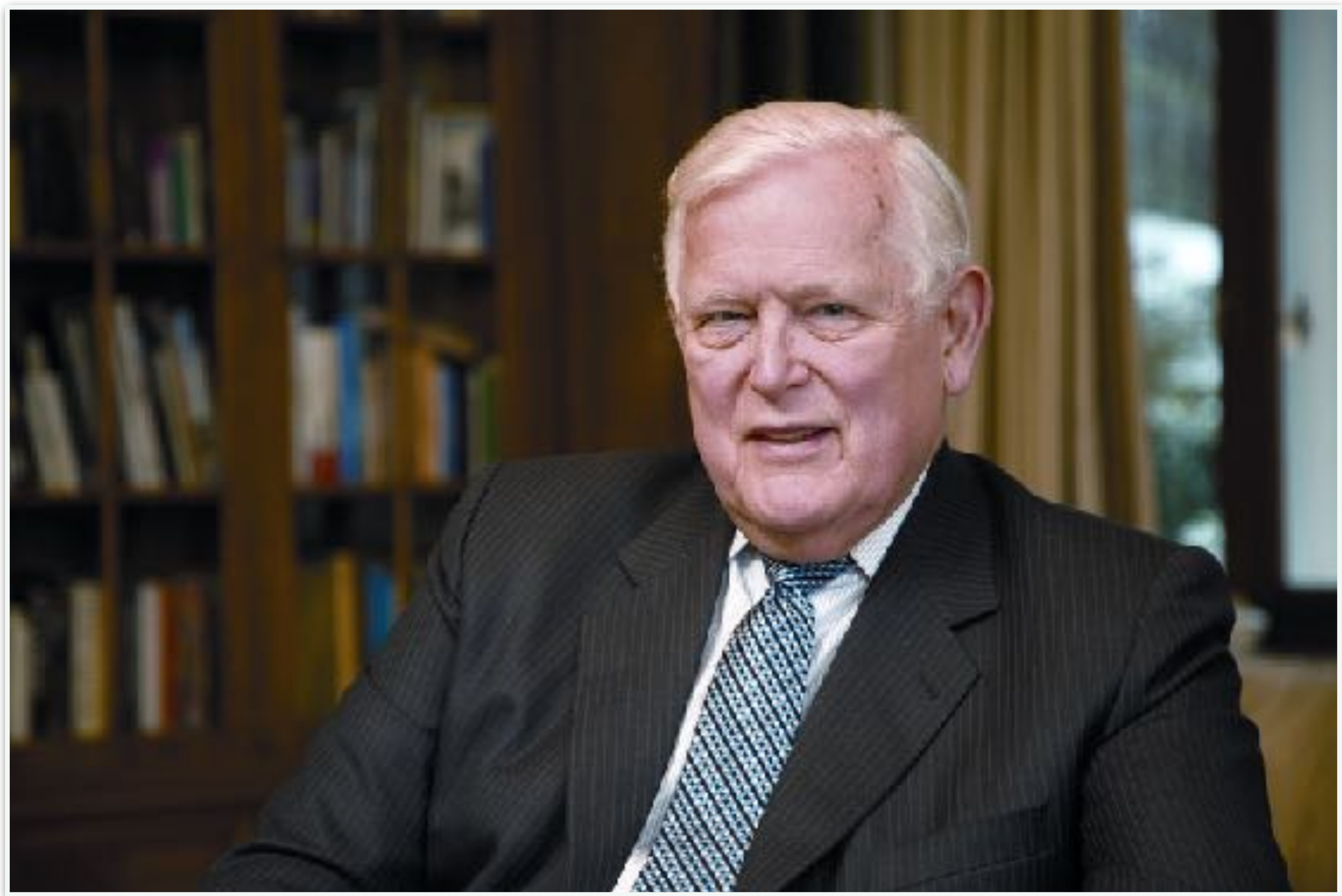
A tribute to

Rudolf E. Kalman



May 19, 1930 - July 2, 2016

CDC 2016
December 11, 2016

by Tryphon T. Georgiou
with gratitude and respect

A tribute to Professor Rudolf E. Kalman

Professor Rudolf Emil Kalman passed away peacefully on July 2, 2016, at his home in Gainesville, Florida. He was 86 years old.

He is survived by his wife Constantina nee Stavrou, their two children Andrew and Elisabeth, eight grandchildren and his brother Otto.

A tribute to Professor Rudolf E. Kalman

Professor Kalman's contributions are timeless and have impacted modern technological and scientific developments across many disciplines.

His thought and style of scientific inquiry have educated countless engineers and scientists.

A tribute to Professor Rudolf E. Kalman

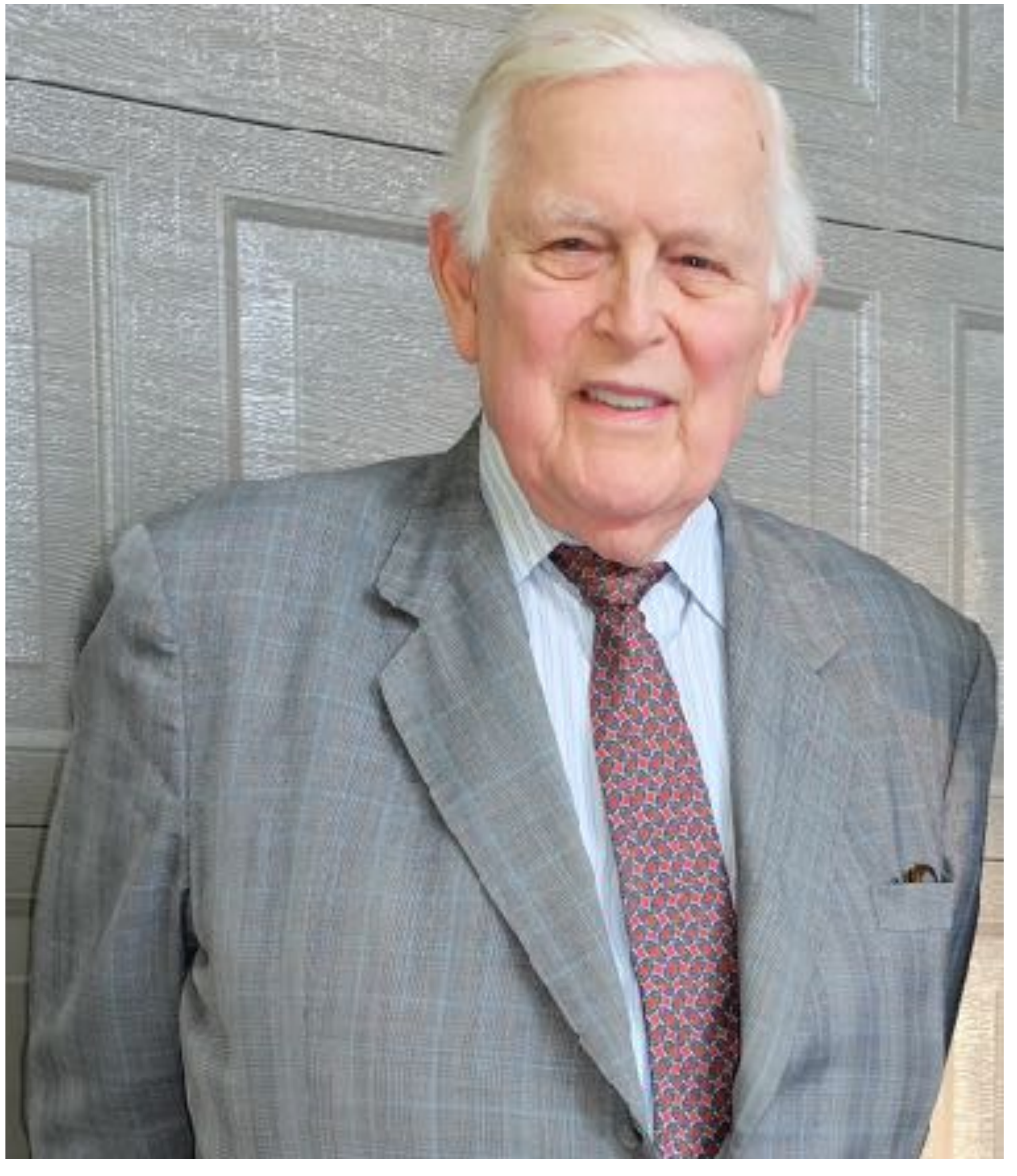
He received numerous awards, including:

IEEE Medal of Honor (1974)
IEEE Centennial Medal (1984)
Kyoto Prize in High Technology from the Inamori Foundation, Japan (1985)
Steele Prize of the American Mathematical Society (1987)
the Bellman Prize (1997)
NAE Charles Stark Draper Prize (2008)

He was a member of:
National Academy of Sciences (USA)
National Academy of Engineering (USA)
American Academy of Arts and Sciences (USA)
numerous foreign Academies

In 2008, he received the **National Medal of Science**, the highest honor the United States gives for scientific achievement.

A tribute to Professor Rudolf E. Kalman



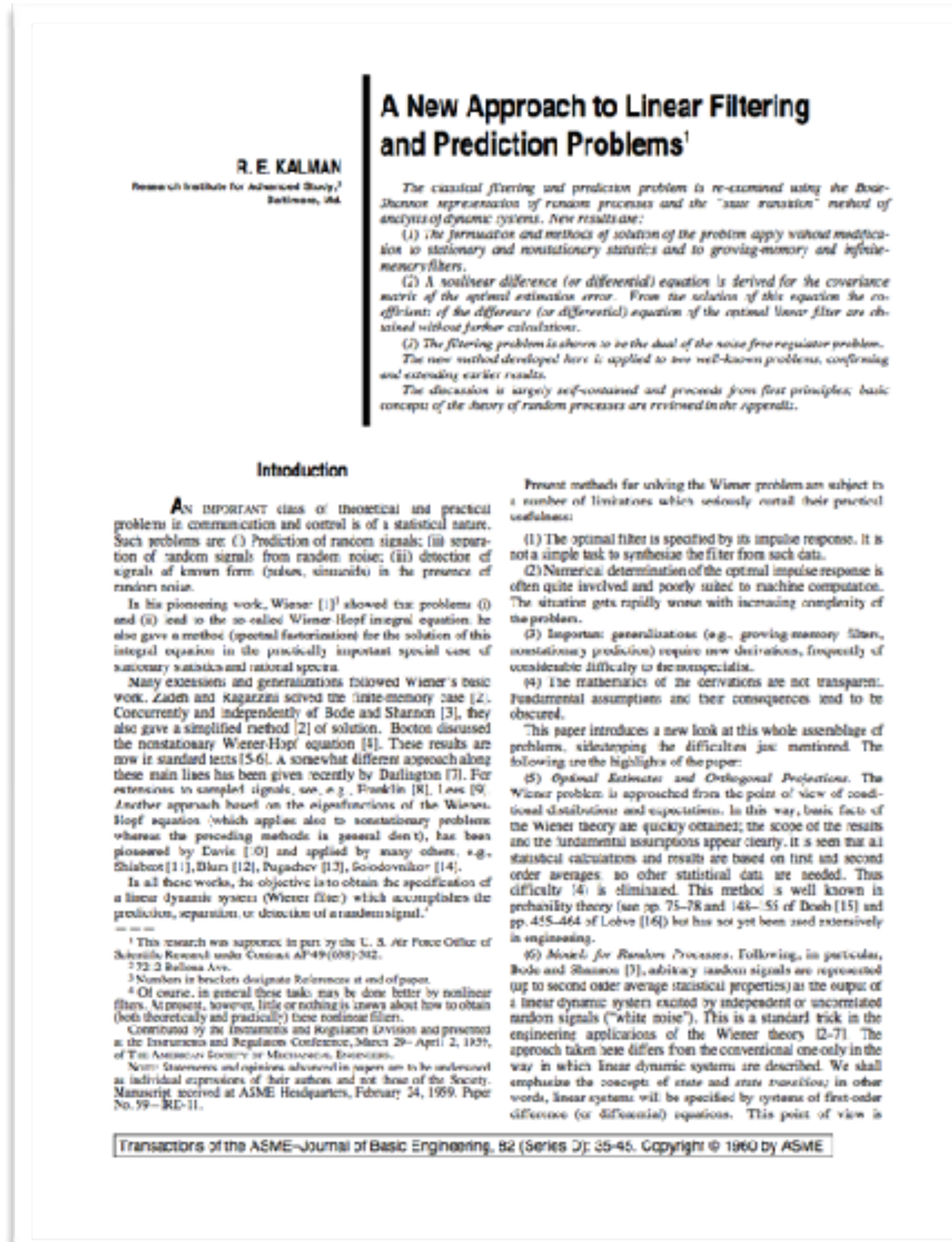
Professor Kalman was a purist in pursuing ideas to completion no matter how long or what effort that necessitated.

His publications were gems, with no exception, in both elegance and scientific depth.

A tribute to Professor Rudolf E. Kalman

A new approach…
20,000+ citations

Kalman, Kalman filtering
~ 1,000,000 citations



A New Approach to Linear Filtering and Prediction Problems[1]

R. E. KALMAN
Research Institute for Advanced Study,[2]
Baltimore, Md.

The classical filtering and prediction problem is re-examined using the Bode-Shannon representation of random processes and the "state transition" method of analysis of dynamic systems. New results are:

(1) The formulation and methods of solution of the problem apply without modification to stationary and nonstationary statistics and to growing-memory and infinite-memory filters.

(2) A nonlinear difference (or differential) equation is derived for the covariance matrix of the optimal estimation error. From the solution of this equation the coefficients of the difference (or differential) equation of the optimal linear filter are obtained without further calculations.

(3) The filtering problem is shown to be the dual of the noise-free regulator problem.

The new method developed here is applied to two well-known problems, confirming and extending earlier results.

The discussion is largely self-contained and proceeds from first principles; basic concepts of the theory of random processes are reviewed in the appendix.

Introduction

An important class of theoretical and practical problems in communication and control is of a statistical nature. Such problems are: (i) Prediction of random signals; (ii) separation of random signals from random noise; (iii) detection of signals of known form (pulses, sinusoids) in the presence of random noise.

In his pioneering work, Wiener [1][3] showed that problems (i) and (ii) lead to the so-called Wiener-Hopf integral equation; he also gave a method (spectral factorization) for the solution of this integral equation in the practically important special case of stationary statistics and rational spectra.

Many extensions and generalizations followed Wiener's basic work. Zadeh and Ragazzini solved the finite-memory case [2]. Concurrently and independently of Bode and Shannon [3], they also gave a simplified method [2] of solution. Booton discussed the nonstationary Wiener-Hopf equation [4]. These results are now in standard texts [5-6]. A somewhat different approach along these main lines has been given recently by Darlington [7]. For extensions to sampled signals, see, e.g., Franklin [8], Lees [9]. Another approach based on the eigenfunctions of the Wiener-Hopf equation (which applies also to nonstationary problems whereas the preceding methods in general don't), has been pioneered by Davis [10] and applied by many others, e.g., Shinbrot [11], Blum [12], Pugachev [13], Solodovnikov [14].

In all these works, the objective is to obtain the specification of a linear dynamic system (Wiener filter) which accomplishes the prediction, separation, or detection of a random signal.[4]

Present methods for solving the Wiener problem are subject to a number of limitations which seriously curtail their practical usefulness:

(1) The optimal filter is specified by its impulse response. It is not a simple task to synthesize the filter from such data.

(2) Numerical determination of the optimal impulse response is often quite involved and poorly suited to machine computation. The situation gets rapidly worse with increasing complexity of the problem.

(3) Important generalizations (e.g., growing-memory filters, nonstationary prediction) require new derivations, frequently of considerable difficulty to the nonspecialist.

(4) The mathematics of the derivations are not transparent. Fundamental assumptions and their consequences tend to be obscured.

This paper introduces a new look at this whole assemblage of problems, sidestepping the difficulties just mentioned. The following are the highlights of the paper:

(5) *Optimal Estimates and Orthogonal Projections.* The Wiener problem is approached from the point of view of conditional distributions and expectations. In this way, basic facts of the Wiener theory are quickly obtained; the scope of the results and the fundamental assumptions appear clearly. It is seen that all statistical calculations and results are based on first and second order averages; no other statistical data are needed. Thus difficulty (4) is eliminated. This method is well known in probability theory (see pp. 75–78 and 148–155 of Doob [15] and pp. 455–464 of Loève [16]) but has not yet been used extensively in engineering.

(6) *Models for Random Processes.* Following, in particular, Bode and Shannon [3], arbitrary random signals are represented (up to second order average statistical properties) as the output of a linear dynamic system excited by independent or uncorrelated random signals ("white noise"). This is a standard trick in the engineering applications of the Wiener theory [2-7]. The approach taken here differs from the conventional one only in the way in which linear dynamic systems are described. We shall emphasize the concepts of *state* and *state transition*; in other words, linear systems will be specified by systems of first-order difference (or differential) equations. This point of view is

[1] This research was supported in part by the U. S. Air Force Office of Scientific Research under Contract AF 49(638)-382.

[2] 7212 Bellona Ave.

[3] Numbers in brackets designate References at end of paper.

[4] Of course, in general these tasks may be done better by nonlinear filters. At present, however, little or nothing is known about how to obtain (both theoretically and practically) these nonlinear filters.

Contributed by the Instruments and Regulators Division and presented at the Instruments and Regulators Conference, March 29– April 2, 1959, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Manuscript received at ASME Headquarters, February 24, 1959. Paper No. 59—IRD-11.

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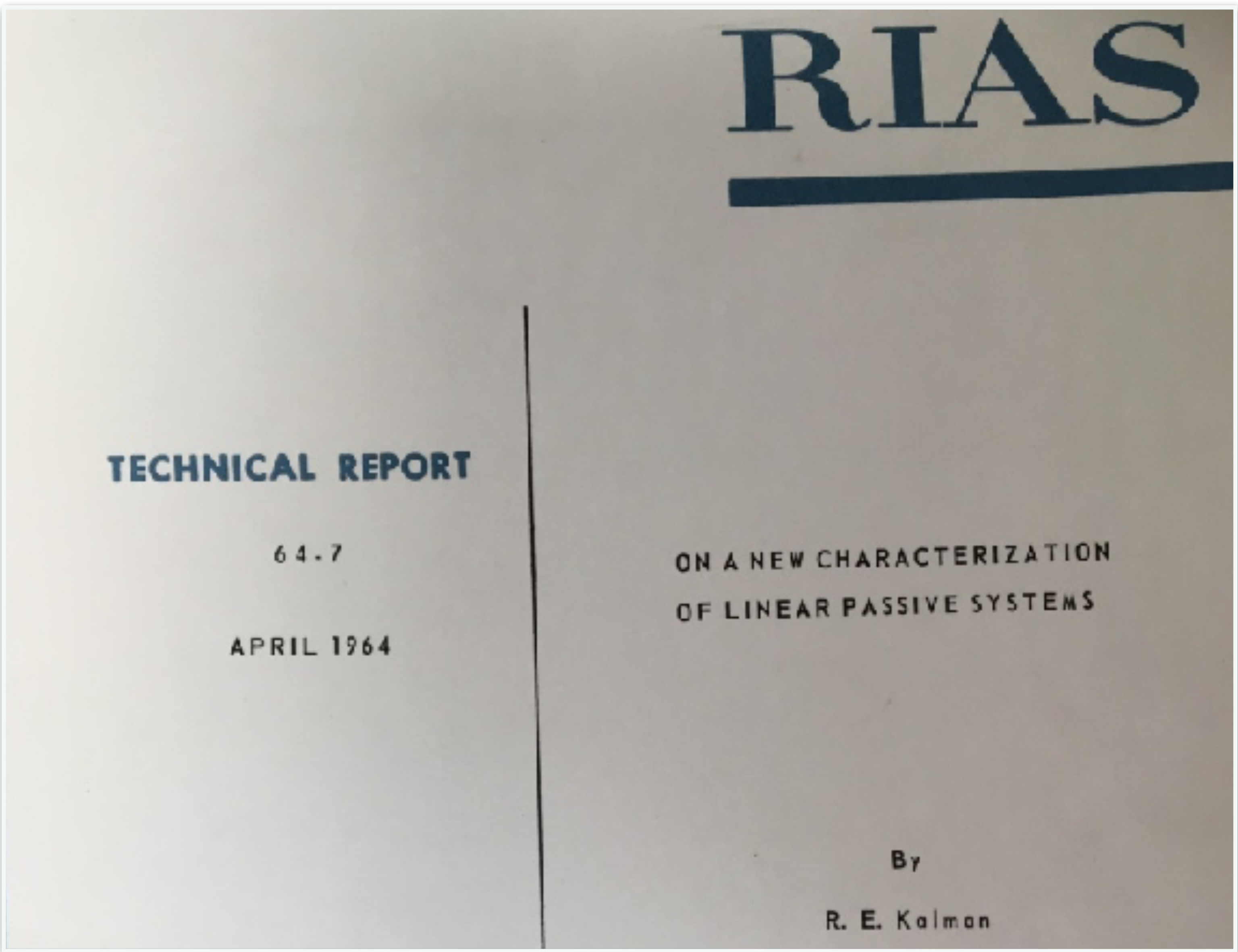

RIAS

TECHNICAL REPORT

64-7

APRIL 1964

ON A NEW CHARACTERIZATION
OF LINEAR PASSIVE SYSTEMS

By

R. E. Kalman

A tribute to Professor Rudolf E. Kalman

BIAS

TECHNICAL REPORT

64-7

APRIL 1964

The Theory of Optimal Control and the Calculus of Variations†

R. E. KALMAN

1. Background

"System theory" today connotes a loose collection of problems and methods held together by a central theme: to understand better the complex systems created by modern technology. Aside from certain combinatorial questions, most of present system theory is concerned with problems in automatic control and in statistical estimation and prediction, with emphasis on solutions that are optimal in some sense. These problems are attacked by a variety of *ad hoc* methods.

Recent research has shown how to formulate and resolve these problems in the spirit of the classical calculus of variations. This provides a unifying point of view. Eventually it should be possible to organize system theory as a rigorous and well-defined discipline. One example of this trend is the author's duality principle (see [1], [2], [3]) relating control and estimation. Conversely, problems in system theory are stimulating further research in the calculus of variations.

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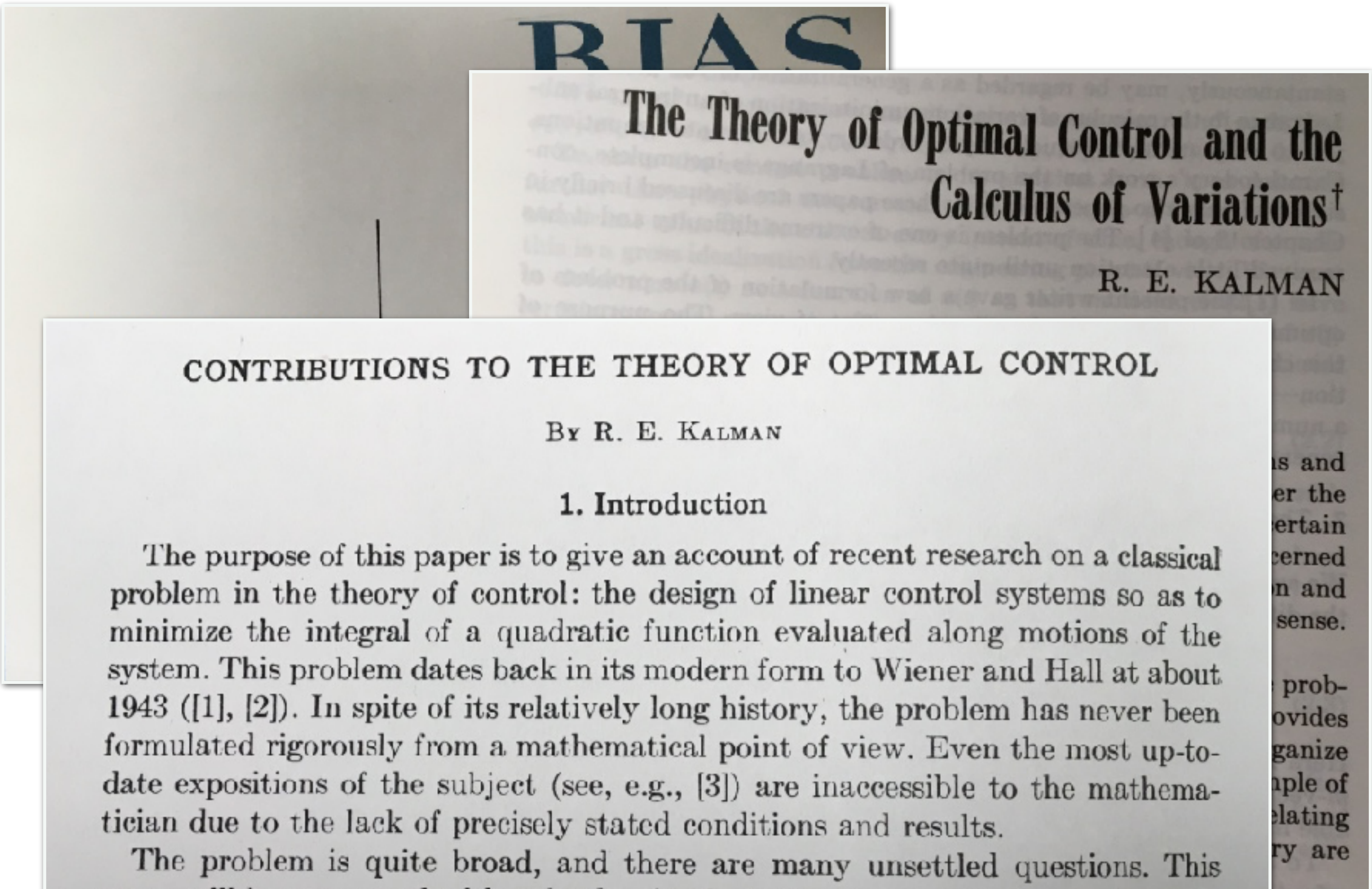

CONTRIBUTIONS TO THE THEORY OF OPTIMAL CONTROL

BY R. E. KALMAN

1. Introduction

The purpose of this paper is to give an account of recent research on a classical problem in the theory of control: the design of linear control systems so as to minimize the integral of a quadratic function evaluated along motions of the system. This problem dates back in its modern form to Wiener and Hall at about 1943 ([1], [2]). In spite of its relatively long history, the problem has never been formulated rigorously from a mathematical point of view. Even the most up-to-date expositions of the subject (see, e.g., [3]) are inaccessible to the mathematician due to the lack of precisely stated conditions and results.

The problem is quite broad, and there are many unsettled questions. This

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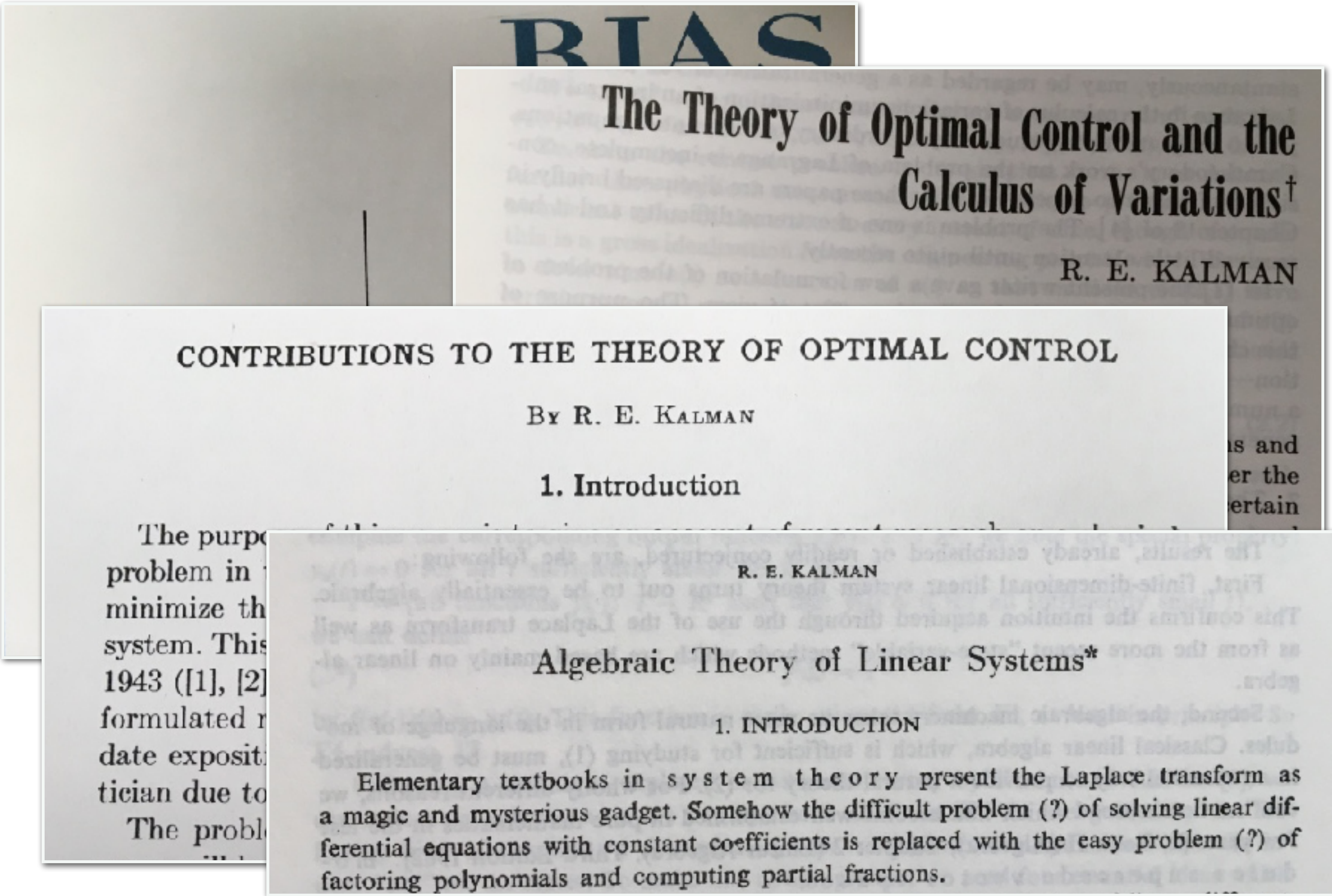

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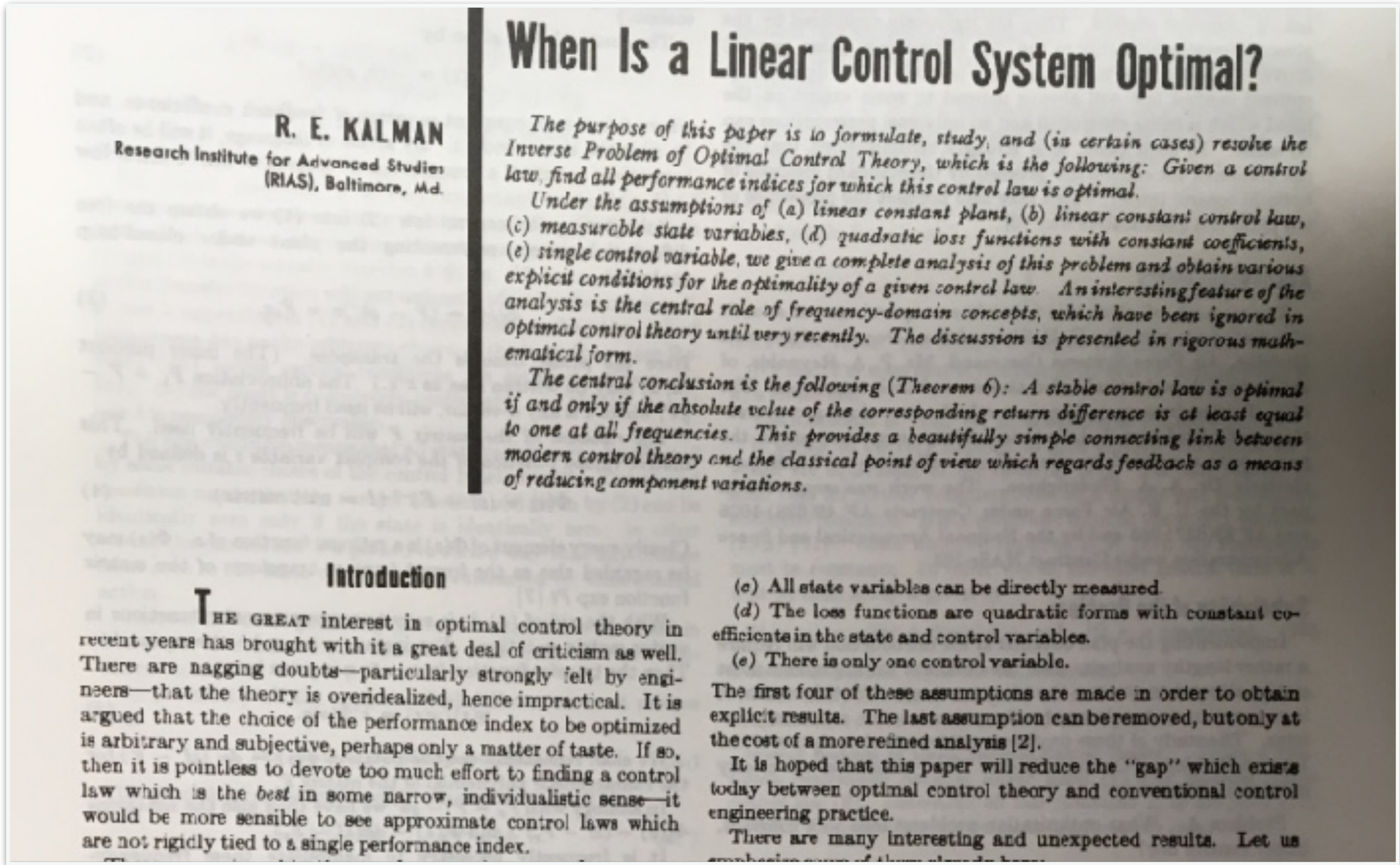

When Is a Linear Control System Optimal?

R. E. KALMAN
Research Institute for Advanced Studies (RIAS), Baltimore, Md.

The purpose of this paper is to formulate, study, and (in certain cases) resolve the Inverse Problem of Optimal Control Theory, which is the following: Given a control law, find all performance indices for which this control law is optimal.

Under the assumptions of (a) linear constant plant, (b) linear constant control law, (c) measurable state variables, (d) quadratic loss functions with constant coefficients, (e) single control variable, we give a complete analysis of this problem and obtain various explicit conditions for the optimality of a given control law. An interesting feature of the analysis is the central role of frequency-domain concepts, which have been ignored in optimal control theory until very recently. The discussion is presented in rigorous mathematical form.

The central conclusion is the following (Theorem 6): A stable control law is optimal if and only if the absolute value of the corresponding return difference is at least equal to one at all frequencies. This provides a beautifully simple connecting link between modern control theory and the classical point of view which regards feedback as a means of reducing component variations.

Introduction

THE GREAT interest in optimal control theory in recent years has brought with it a great deal of criticism as well. There are nagging doubts—particularly strongly felt by engineers—that the theory is overidealized, hence impractical. It is argued that the choice of the performance index to be optimized is arbitrary and subjective, perhaps only a matter of taste. If so, then it is pointless to devote too much effort to finding a control law which is the *best* in some narrow, individualistic sense—it would be more sensible to see approximate control laws which are not rigidly tied to a single performance index.

(*c*) All state variables can be directly measured.

(*d*) The loss functions are quadratic forms with constant coefficients in the state and control variables.

(*e*) There is only one control variable.

The first four of these assumptions are made in order to obtain explicit results. The last assumption can be removed, but only at the cost of a more refined analysis [2].

It is hoped that this paper will reduce the "gap" which exists today between optimal control theory and conventional control engineering practice.

There are many interesting and unexpected results. Let us

View your subject from all possible angles

1971 -
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Library of the Center for Mathematical Systems Theory
Gainesville, Florida

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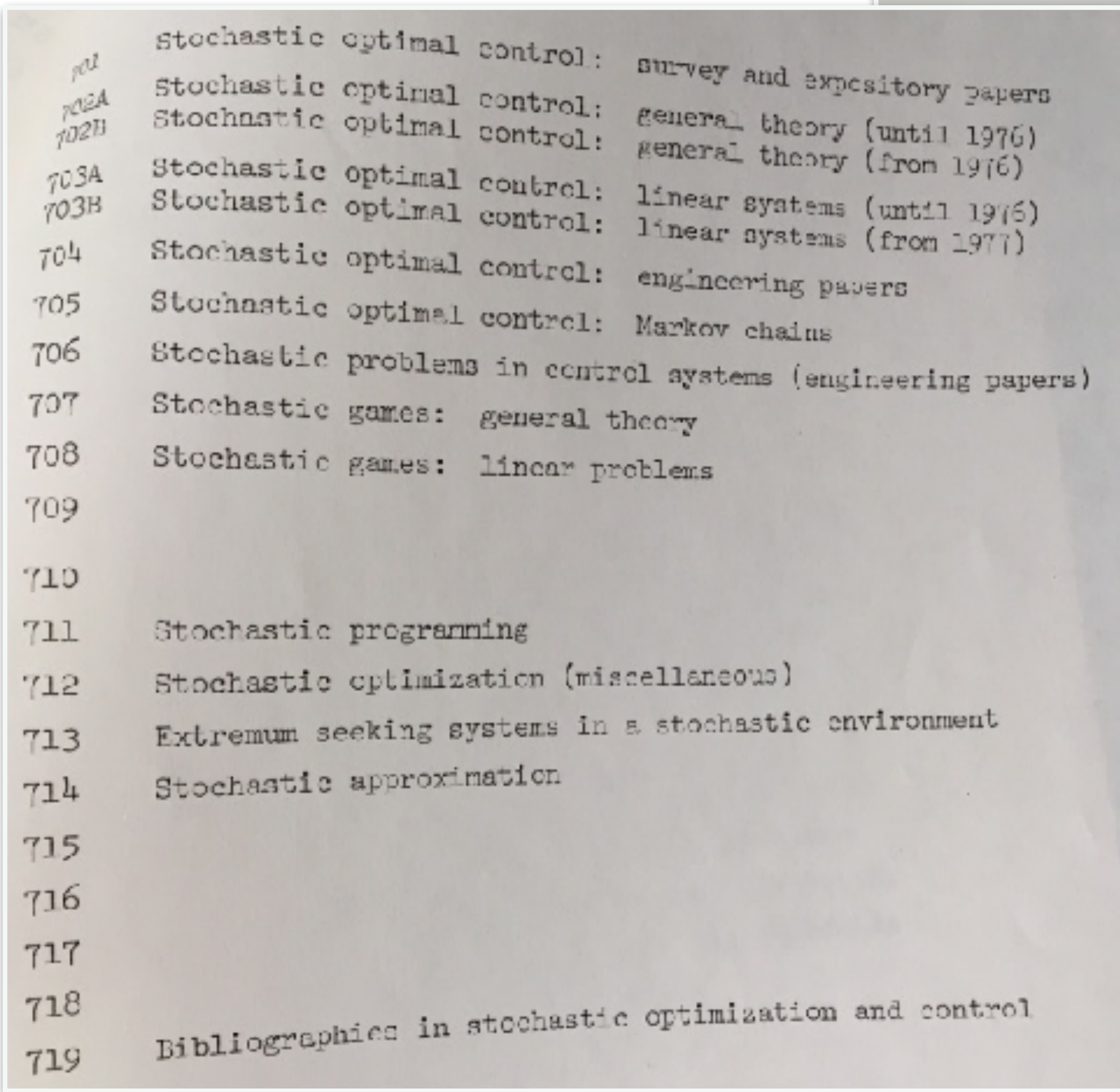

701 Stochastic optimal control: survey and expository papers
702A Stochastic optimal control: general theory (until 1976)
702B Stochastic optimal control: general theory (from 1976)
703A Stochastic optimal control: linear systems (until 1976)
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708 Stochastic games: linear problems
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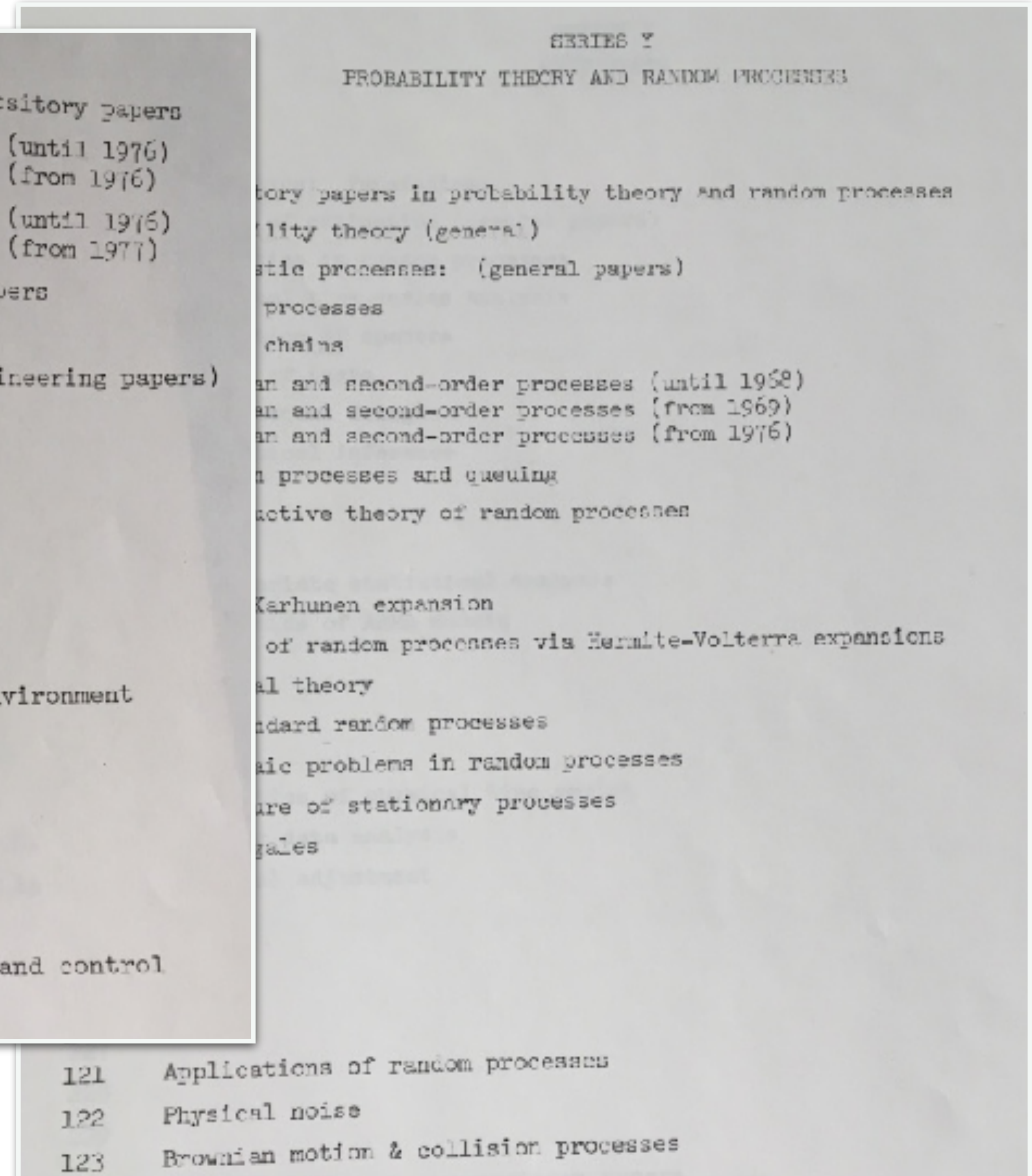

SERIES 1
PROBABILITY THEORY AND RANDOM PROCESSES

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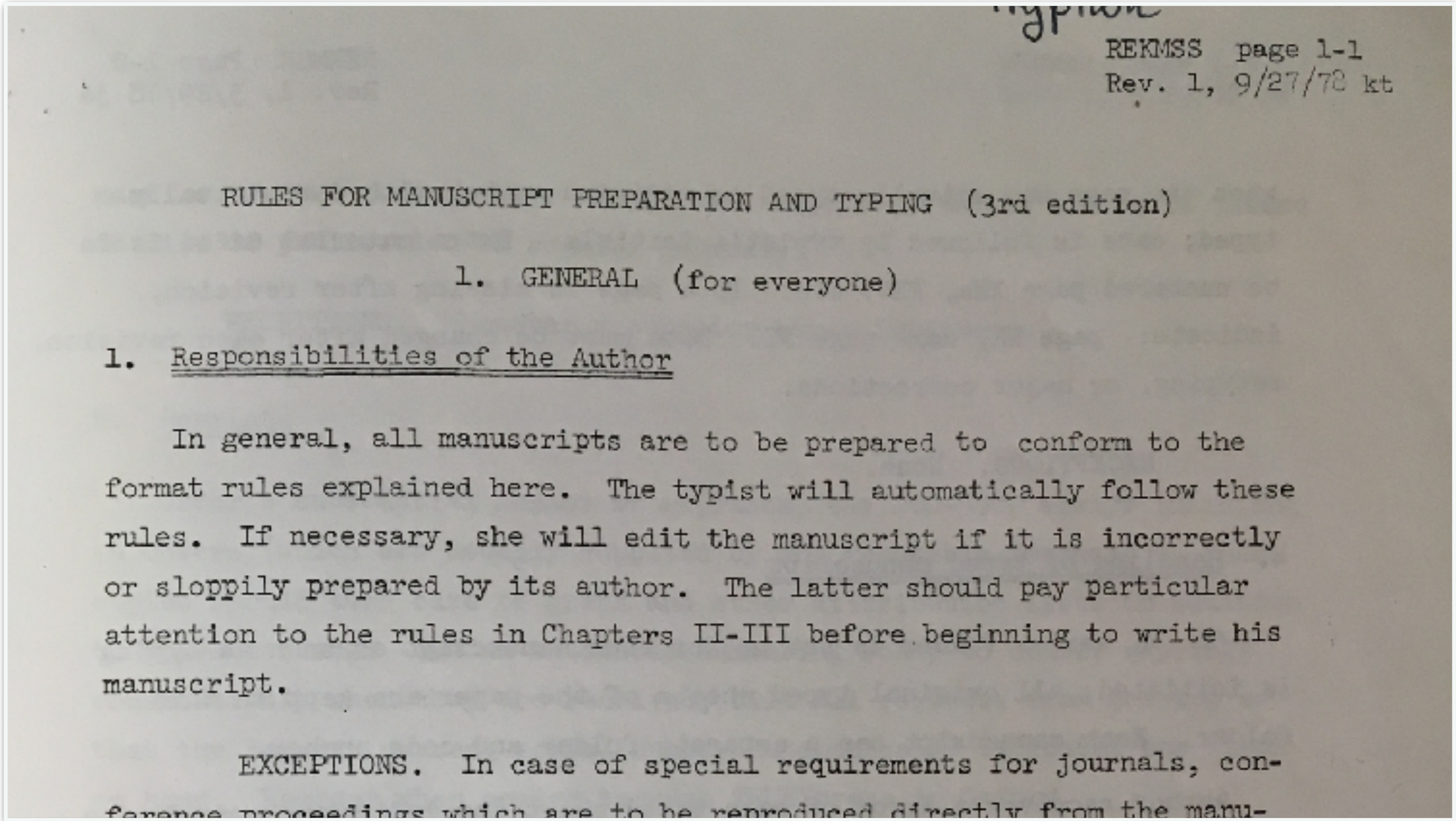
A tribute to Professor Rudolf E. Kalman



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1. GENERAL (for everyone)

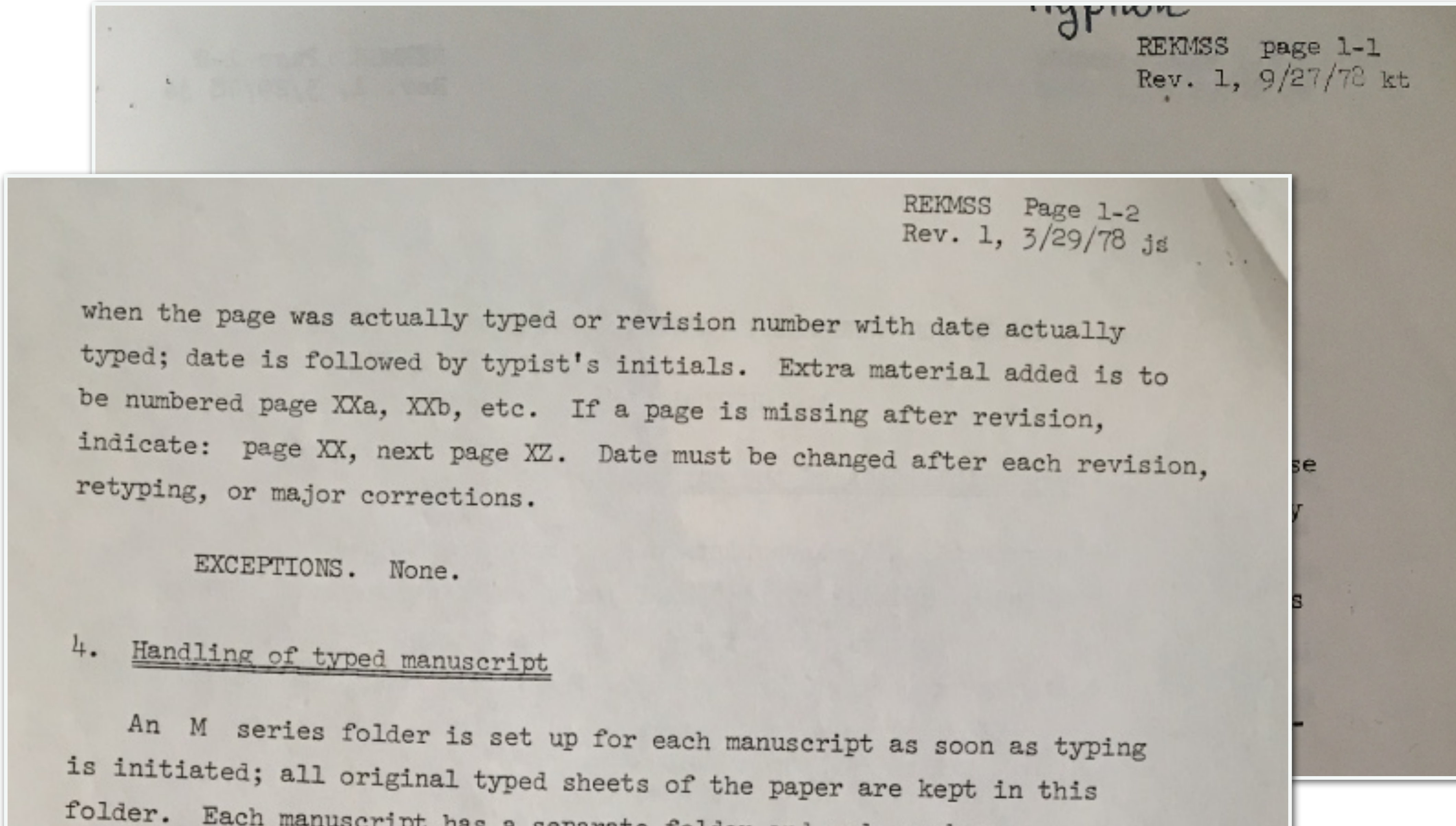
1. Responsibilities of the Author

In general, all manuscripts are to be prepared to conform to the format rules explained here. The typist will automatically follow these rules. If necessary, she will edit the manuscript if it is incorrectly or sloppily prepared by its author. The latter should pay particular attention to the rules in Chapters II-III before beginning to write his manuscript.

EXCEPTIONS. In case of special requirements for journals, conference proceedings which are to be reproduced directly from the manu-

Rules and precision
“Latex before Latex”

A tribute to Professor Rudolf E. Kalman



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when the page was actually typed or revision number with date actually typed; date is followed by typist's initials. Extra material added is to be numbered page XXa, XXb, etc. If a page is missing after revision, indicate: page XX, next page XZ. Date must be changed after each revision, retyping, or major corrections.

EXCEPTIONS. None.

4. Handling of typed manuscript

An M series folder is set up for each manuscript as soon as typing is initiated; all original typed sheets of the paper are kept in this folder. Each manuscript has a separate folder and ...

Rules and precision
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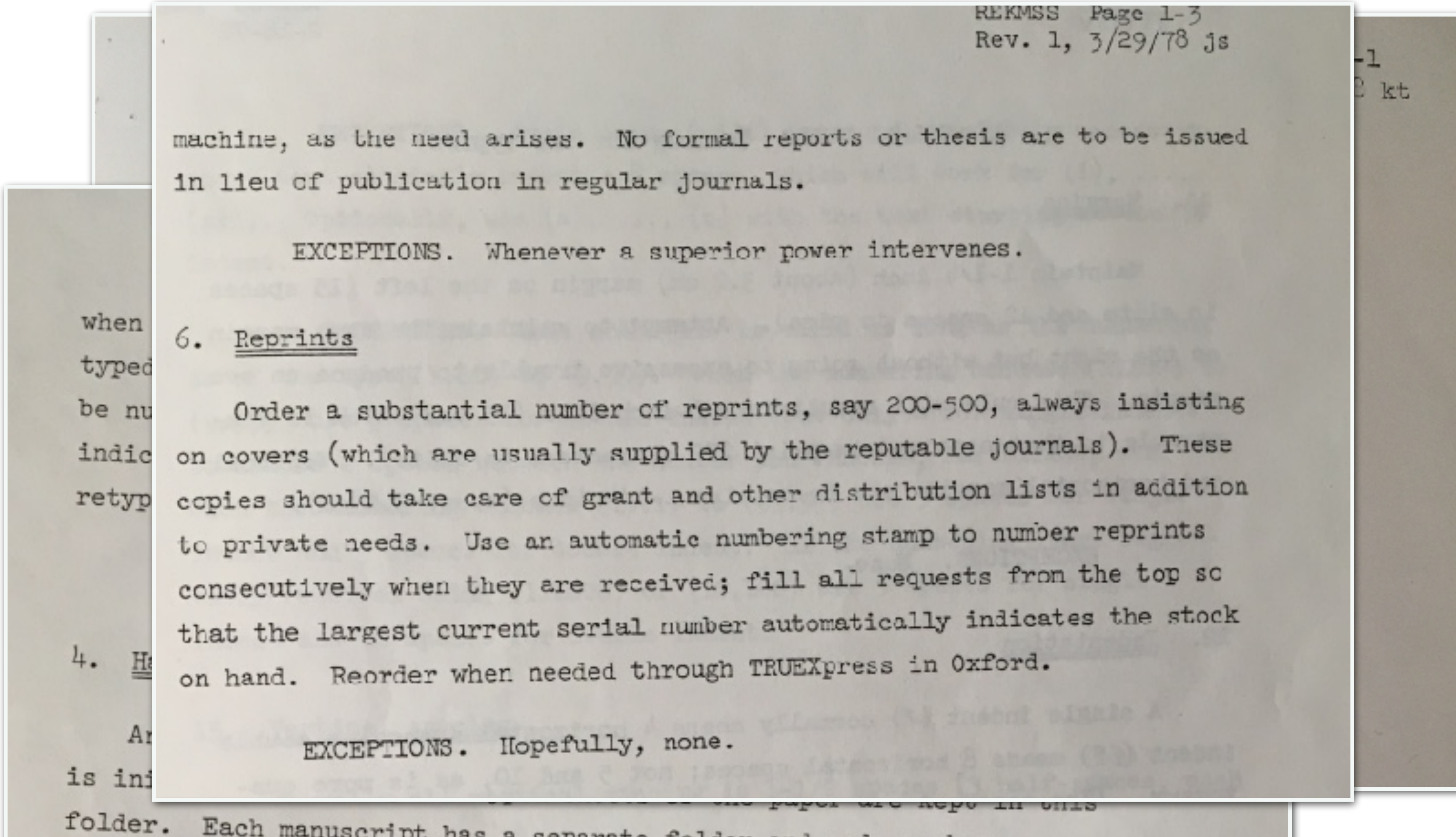
A tribute to Professor Rudolf E. Kalman



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machine, as the need arises. No formal reports or thesis are to be issued in lieu of publication in regular journals.

EXCEPTIONS. Whenever a superior power intervenes.

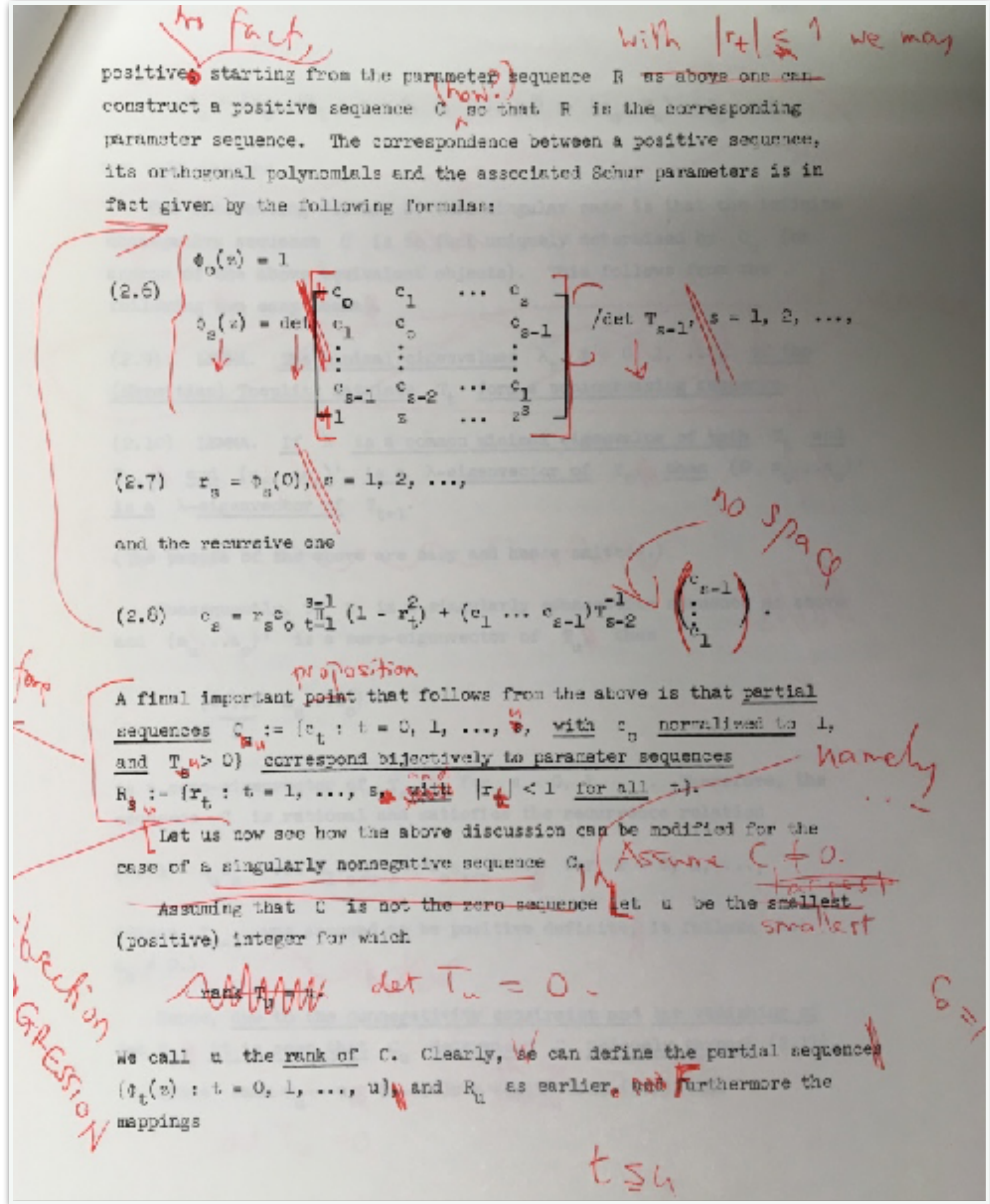
6. Reprints

Order a substantial number of reprints, say 200-500, always insisting on covers (which are usually supplied by the reputable journals). These copies should take care of grant and other distribution lists in addition to private needs. Use an automatic numbering stamp to number reprints consecutively when they are received; fill all requests from the top so that the largest current serial number automatically indicates the stock on hand. Reorder when needed through TRUEXpress in Oxford.

EXCEPTIONS. Hopefully, none.

Rules and precision
“Latex before Latex”

Being correct is only
a necessary condition



positive, starting from the parameter sequence R as above one can construct a positive sequence C so that R is the corresponding parameter sequence. The correspondence between a positive sequence, its orthogonal polynomials and the associated Schur parameters is in fact given by the following formulas:

$$\text{(2.6)}\quad \begin{cases} \phi_0(z) = 1 \\ \phi_s(z) = \det \begin{bmatrix} c_0 & c_1 & \cdots & c_s \\ c_1 & c_0 & & c_{s-1} \\ \vdots & \vdots & & \vdots \\ c_{s-1} & c_{s-2} & \cdots & c_1 \\ 1 & z & \cdots & z^s \end{bmatrix} / \det T_{s-1}, \quad s = 1, 2, \ldots, \end{cases}$$

$$\text{(2.7)}\quad r_s = \phi_s(0), \quad s = 1, 2, \ldots,$$

and the recursive one

$$\text{(2.8)}\quad c_s = r_s c_0 \prod_{t=1}^{s-1} (1 - r_t^2) + (c_1 \cdots c_{s-1}) T_{s-2}^{-1} \begin{pmatrix} c_{s-1} \\ \vdots \\ c_1 \end{pmatrix}.$$

A final important point that follows from the above is that partial sequences $C_s := \{c_t : t = 0, 1, \ldots, s$, with c_0 normalized to 1, and $T_s > 0\}$ correspond bijectively to parameter sequences $R_s := \{r_t : t = 1, \ldots, s$, with $|r_t| < 1$ for all $t\}$.

Let us now see how the above discussion can be modified for the case of a singularly nonnegative sequence C.

Assuming that C is not the zero sequence let u be the smallest (positive) integer for which

rank T_u = u.

We call u the rank of C. Clearly, we can define the partial sequences $\{\phi_t(z) : t = 0, 1, \ldots, u\}$ and R_u as earlier, and furthermore the mappings

corrections by REK to a student (ttg) manuscript

A tribute to Professor Rudolf E. Kalman

Gainesville, 1983

A tribute to Professor Rudolf E. Kalman

Gainesville, June 12, 2016

A tribute to Professor Rudolf E. Kalman

"*My best wishes for the conference…[mtns 2016]. It is the kind of work Rudy devoted his life to.*"

Dina Kalman
July 11, 2016